# Strength Design of Masonry

October 12, 2017

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The Masonry Society

AIA Provider: 50119857

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# Course Description

Strength design was added to the TMS 402 Building Code for Masonry Structures in 2002. However, most masonry is still designed by the Allowable Stress Design method. Strength design generally results in more efficient designs than with Allowable Stress Design. This webinar will review the design assumptions for strength design, and look at the design of beams, bearing walls, and shear walls using strength design. Practical design methods and tips will be provided for each member type, and examples will illustrate the design process. The results of each design will be compared to allowable stress design. This webinar will provide the engineer with the tools necessary to begin using strength design for masonry, and realizing the benefits it can offer.

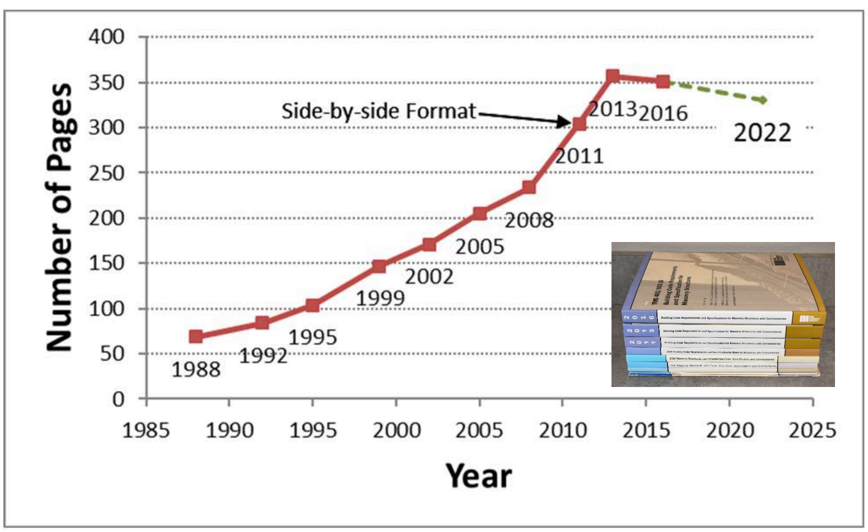


# Learning Objectives

- Understand the assumptions and basis for strength design of masonry.
- Understand the design methods appropriate for masonry beams, and be able to design a beam using strength design.
- Understand the design methods appropriate for masonry bearing walls under out-of-plane loads, and be able to design a bearing wall using strength design.
- Understand the design methods appropriate for shear walls, including both shear and overturning, and be able to design a shear wall using strength design.

## Summary of Major Code Changes

- 2011
  - Recalibration of allowable stresses due to removal of 1/3 stress increase
  - ASD shear provisions similar to SD
- 2013
  - Increase in $f'_m$ to 2000 psi for Type S mortar
  - Moment magnification for out-of-plane loading
  - Reorganization: SD moved to Chapter 9
- 2016
  - Shear friction provisions
  - Anchor bolt strength



## TMS 402 Chapter 9
### 9.1 General

- 9.1.1 Scope
- 9.1.2 Required strength
- 9.1.3 Design strength
- 9.1.4 Strength-reduction factors
- 9.1.5 Deformation requirements
- 9.1.6 Anchor bolts embedded in grout
- 9.1.7 Shear strength in multiwythe elements
- 9.1.8 Nominal bearing strength
- 9.1.9 Material properties

## Design Strength: Strength Reduction Factors

| Action | Reinforced Masonry | Unreinforced Masonry |
|---|---|---|
| combinations of flexure and axial load | 0.90 | 0.60 |
| shear | 0.80 | |
| bearing | 0.60 | |
| anchor bolts: pryout | 0.50 | |
| anchor bolts: controlled by anchor bolt steel | 0.90 | |
| anchor bolts: pullout | 0.65 | |

## Table 9.1.9.2 Modulus of Rupture

| Masonry Type | Mortar Type | | | |
|---|---|---|---|---|
| | Portland cement/lime or mortar cement | | Masonry Cement | |
| | M or S | N | M or S | N |
| **Normal to Bed Joints** | | | | |
| Solid Units | 133 | 100 | 80 | 51 |
| Hollow Units[1] | | | | |
| Ungrouted | 84 | 64 | 51 | 31 |
| Fully Grouted | 163 | 158 | 153 | 145 |
| **Parallel to bed joints in running bond** | | | | |
| Solid Units | 267 | 200 | 160 | 100 |
| Hollow Units | | | | |
| Ungrouted and partially grouted | 167 | 127 | 100 | 64 |
| Fully grouted | 267 | 200 | 160 | 100 |
| **Parallel to bed joints not laid in running bond** | | | | |
| Continuous grout section parallel to bed joints | 335 | 335 | 335 | 335 |
| Other | 0 | 0 | 0 | 0 |

[1] Linear interpolate for partial grout

## SD of Unreinforced Masonry: TMS 402 Section 9.2

- 9.2.1 Scope
- 9.2.2 Design criteria
- 9.2.3 Design assumptions
- 9.2.4 Nominal flexural and axial strength
- 9.2.5 Axial tension
- 9.2.6 Nominal shear strength

Key design equation: $f_t = \frac{Mc}{I} - \frac{P}{A}$

## SD of Reinforced Masonry: TMS 402 Chapter 9

- 9.3.1 Scope
- 9.3.2 Design assumptions
- 9.3.3 Reinforcement requirements and details, including maximum steel percentage
- 9.3.4 Design of beams and columns
  - nominal axial and flexural strength
  - nominal shear strength
- 9.3.5 Wall design for out – of – plane loads
- 9.3.6 Wall design for in – plane loads

## Design Assumptions TMS 402 Section 9.3.2

- continuity between reinforcement and grout
- $\varepsilon_{mu}$ = 0.0035 for clay masonry
  $\varepsilon_{mu}$ = 0.0025 for concrete masonry
- plane sections remain plane
- elasto – plastic stress – strain curve for reinforcement
- tensile strength of masonry is neglected
- equivalent rectangular compressive stress block:
  - stress = 0.80 $f'_m$
  - depth $a$ = 0.80$c$

## Reinforcement:

- Size Limitations (9.3.3.1)
  - Maximum bar size is #9
  - Bar diameter ≤ 1/8 nominal wall thickness (6.1.2.5)
  - Bar diameter ≤ ¼ least clear dimension of cell
  - Area ≤ 4% of cell area (8% at splices)
- Shear Reinforcement (6.1.7.1)
  - Bend around edge reinforcement with a 180° hook
  - At wall intersections, bend around edge reinforcement with a 90° hook and extend horizontally into intersecting wall a minimum of development length
- Bars not allowed to be bundled (9.3.3.3)

## Flexural Members

- **Beams and Lintels**
- **Non-bearing walls**

## Beams and Lintels: TMS 402 Sec. 9.3.4.2

**Minimum reinforcement:** (9.3.4.2.2.2, 9.3.4.2.2.3)

- $M_n \geq 1.3$ x cracking strength
  - or $A_s \geq (4/3)A_{s,req'd}$
- Modulus or rupture, $f_r$ = Table 9.1.9.2

**Maximum reinforcement:** (9.3.3.2)

$$\rho_{\max} = \frac{0.8(0.8)f'_m}{f_y}\left(\frac{\varepsilon_m}{\varepsilon_m + \varepsilon_s}\right)$$

$\varepsilon_s = 1.5\varepsilon_y$

| | Grade 60 steel | |
|---|---|---|
| | Clay | CMU |
| $\rho_{max}$ | $0.00565f'_m$<br>$0.843\rho_{bal}$ | $0.00476f'_m$<br>$0.815\rho_{bal}$ |
| $\rho_{max}$<br>$f'_m$ = 2 ksi | 0.01131 | $\boxed{0.00952}$ |

$f'_m$ in ksi

## Beams and Lintels: Design

Nominal Moment Strength

$$M_n = A_s f_y\left(d - \frac{1}{2}\frac{A_s f_y}{0.8 f'_m b}\right)$$

Nominal Shear Strength

$$V_{nm} = 2.25A_{nv}\sqrt{f'_m}$$

$$V_{ns} = 0.5\left(\frac{A_v}{s}\right)f_y d_v$$

$$V_n \leq 4A_{nv}\sqrt{f'_m}$$

**Design Procedure**

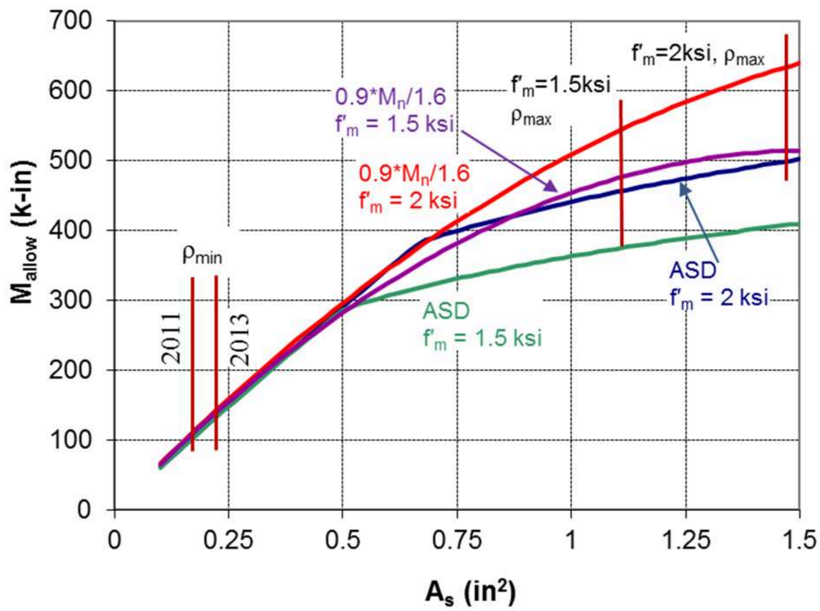
1. Determine *a*, depth of compressive stress block

$$a = d - \sqrt{d^2 - \frac{2M_n}{0.8 f'_m b}} = d - \sqrt{d^2 - \frac{2M_u}{0.8\phi f'_m b}}$$

2. Solve for $A_s$

$$A_s = \frac{0.8 f'_m ba}{f_y}$$

$\phi$ = 0.9 flexure $\phi$ = 0.8 shear




# Example: Beam

<u>Given:</u> 10 ft. opening; dead load of 1.5 kip/ft; live load of 1.5 kip/ft; 24 in. high; Grade 60 steel; Type S masonry cement mortar; 8 in. CMU; $f'_m$ = 2000 psi
<u>Required:</u> Design beam
<u>Solution:</u>

**5.2.1.3:** Length of bearing of beams shall be a minimum of 4 in.; typically assumed to be 8 in.

**5.2.1.1.1** Span length of members not built integrally with supports shall be taken as the clear span plus depth of member, but need not exceed distance between center of supports.

- Span = 10 ft + 2(4 in.) = 10.67 ft

**5.2.1.2** Compression face of beams shall be laterally supported at a maximum spacing of:

- 32 multiplied by the beam thickness. 32(7.625 in.) = 244 in. = 20.3 ft
- $120b^2/d$. 120(7.625 in.)$^2$ / (20 in.) = 349 in. = 29.1 ft

## Strength Design: Flexure

**Factored Load**
Weight of fully grouted normal weight: 83 psf

$$w_u = 1.2D + 1.6L = 1.2\left(1.5\tfrac{k}{ft} + 0.083\tfrac{k}{ft^2}(2\,ft)\right) + 1.6\left(1.5\tfrac{k}{ft}\right) = 4.40\tfrac{k}{ft}$$

**Factored Moment**

$$M_u = \frac{w_u L^2}{8} = \frac{\left(4.40\tfrac{k}{ft}\right)(10.67\,ft)^2}{8} = 62.6k-ft$$

**Find $a$**
Depth of equivalent rectangular stress block

$$a = d - \sqrt{d^2 - \frac{2M_n}{0.8f'_m b}} = 20in - \sqrt{(20in)^2 - \frac{2\left(\frac{62.6k-ft}{0.9}\right)\left(\frac{12in}{ft}\right)}{0.8(2.0ksi)(7.625in)}} = 3.78in$$

**Find $A_s$**
Area of steel

$$A_s = \frac{0.8f'_m ba}{f_y} = \frac{0.8(2.0ksi)(7.625in)(3.78in)}{60ksi} = 0.77in^2$$

$\boxed{\text{Use 2 - \#6 } (A_s = 0.88\ in^2)}$

$M_n$ = 78.5 k-ft
$\phi M_n$ = 70.6 k-ft

## Check Min and Max Reinforcement

**Minimum Reinforcement Check:** $f_r$ = 160 psi (parallel to bed joints in running bond; fully grouted)

Section modulus
$$S_n = \frac{bh^2}{6} = \frac{(7.625in)(24in)^2}{6} = 732in^3$$

Cracking moment
$$M_{cr} = S_n f_r = (732in^3)(160\,psi) = 117.1k-in = 9.76k-ft$$

Check $1.3M_{cr}$
$$1.3M_{cr} = 1.3(9.76k-ft) = 12.7k-ft \quad \le \quad M_n = 75.3k-ft$$

**Maximum Reinforcement Check:**

$f'_m$ = 2 ksi $\quad \rho_{max} = 0.00952$

$$\rho = \frac{A_s}{bd} = \frac{0.88in^2}{(7.625in)(20in)} = 0.00577$$

## Summary: Beams, Flexure

| Dead Load (k/ft) (superimposed) | Live Load (k/ft) | Required $A_s$ (in$^2$) ASD | SD |
|---|---|---|---|
| 0.5 | 0.5 | 0.34<br>0.34 ($f'_m$ = 1.5 ksi) | 0.26<br>0.26 ($f'_m$ = 1.5 ksi) |
| 1.0 | 1.0 | 0.64<br>0.65 ($f'_m$ = 1.5 ksi) | 0.50<br>0.52 ($f'_m$ = 1.5 ksi) |
| 1.5 | 1.5 | 1.94<br>5.09 ($f'_m$ = 1.5 ksi) | 0.77<br>0.80 ($f'_m$ = 1.5 ksi) |

ASD: Allowable tension controls for 0.5 k/ft and 1 k/ft.

## Strength Design: Shear

Shear at reaction

$$V_u = \frac{w_u L}{2} = \frac{\left(1.2\left(1.5\,\tfrac{k}{ft} + 0.083\,\tfrac{k}{ft^2}(2\,ft)\right) + 1.6\left(1.5\,\tfrac{k}{ft}\right)\right)(10.67\,ft)}{2} = 23.47k$$

*d*/2 from face of support

$$\left(\frac{20in.}{2} + 4in.\right)\frac{1\,ft}{12in.} = 1.17\,ft$$

Requirement for shear at *d*/2 from face of support is in ASD, not SD, but assume it applies

Design shear force

$$V_u = 23.47k\left(\frac{5.33\,ft - 1.17\,ft}{5.33\,ft}\right) = 18.33k$$

Suggest that *d* be used, not $d_v$ to find $A_{nv}$

Design masonry shear strength

$$\phi V_{nm} = \phi(2.25)A_{nv}\sqrt{f'_m}$$
$$= 0.8(2.25)(7.625in.)(20in.)\sqrt{2000\,psi} = 12.28k$$

## Strength Design: Shear

Check max $V_n$ $\phi V_n \le \phi 4 A_{nv}\sqrt{f'_m} = 0.8(4)(7.625in.)(20in.)\sqrt{2000psi} = 21.82k$

> 18.33*k* OK

Req'd $V_{ns}$ $V_{ns} = \frac{V_u - \phi V_{nm}}{\phi} = \frac{18.33 - 12.27}{0.8} = 7.58kips$

Determine $A_v$ for a spacing of 8 in.

$$V_{ns} = 0.5\left(\frac{A_v}{s}\right) f_y d_v \Rightarrow A_v = \frac{V_{ns} s}{0.5 f_y d_v}$$

$$A_v = \frac{7.58k(8in.)}{0.5(60ksi)(20in.)} = 0.101in^2$$

**Use #3 stirrups**

## Combined Bending and Axial Load

- **Columns**
- **Pilasters**

## Combined Bending and Axial Load: TMS 402 Sec. 9.3.4.1.1

- $P_n$ and $M_n$ determined using design assumptions of Section 9.3.2
  - Construct interaction diagram
- Nominal axial strength limited by

$$P_n = 0.80\left[0.80 f'_m (A_n - A_{st}) + f_y A_{st}\right]\left[1 - \left(\frac{h}{140r}\right)^2\right] \quad \text{for } \frac{h}{r} \le 99 \quad (9\text{-}15)$$

$$P_n = 0.80\left[0.80 f'_m (A_n - A_{st}) + f_y A_{st}\right]\left(\frac{70r}{h}\right)^2 \quad \text{for } \frac{h}{r} > 99 \quad (9\text{-}16)$$

Compressive strength of reinforcement is ignored unless the reinforcement is tied in compliance with TMS 402 Section 5.3.1.4

## Combined Bending and Axial Load: Design Method

Calculate

$$a = d - \sqrt{d^2 - \frac{2\left[P_u(d - t/2) + M_u\right]}{\phi(0.8 f'_m b)}} \qquad c = \frac{a}{0.8}$$

Is $c \ge c_{bal}$?
For Grade 60 steel
$c_{bal} = 0.547d$

$$c_{bal} = \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \varepsilon_y} d$$

YES

$$A_s = \frac{0.8 f'_m ba - P_u/\phi}{\varepsilon_{mu} E_s \left(\frac{d - c}{c}\right)}$$

Compression controls

NO

$$A_s = \frac{0.8 f'_m ba - P_u/\phi}{f_y}$$

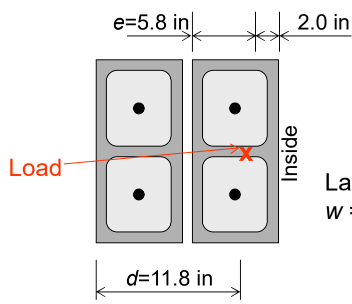
Tension controls

## Example: Pilaster Design

Given: Nominal 16 in. wide x 16 in. deep CMU pilaster; $f'_m$ = 2000 psi; Grade 60 bar in each corner, center of cell; Effective height = 24 ft; Dead load of 9.6 kips and snow load of 9.6 kips act at an eccentricity of 5.8 in. (2 in. inside of face); Factored wind load of 26 psf (pressure and suction) and uplift of 8.1 kips (e=5.8 in.); Pilasters spaced at 16 ft on center; Wall is assumed to span horizontally between pilasters; No ties.

Required: Determine required reinforcing using strength design.

Solution:

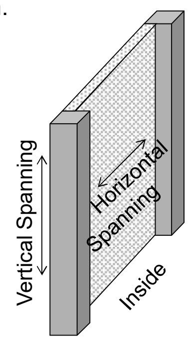



$E_m$ = 1800ksi

$n$ = 16.1

Lateral Load

$w$ = 0.6(26psf)(16ft)=250 lb/ft




## Example: Pilaster Design

**0.9D + 1.0W**

At top of pilaster:

$$P_u = 0.9(9.6k) + 1.0(-8.1k) = 0.54k$$

$$M_u = P_u e = 0.54k(5.8in.) = 3.1k - in$$

Find location of maximum moment

$$x = \frac{h}{2} - \frac{M}{wh} = \frac{288in}{2} - \frac{3.1kip - in}{0.416\frac{k}{ft}(24\,ft)} = 143.7in$$

$$M_u = \frac{M}{2} + \frac{wh^2}{8} + \frac{M^2}{2wh^2} = \frac{3.1k - in}{2} + \frac{0.0347\frac{k}{in}(288in)^2}{8} + \frac{(3.1k - in)^2}{2\left(0.0347\frac{k}{in}\right)(288in)^2} = 361.0k - in$$

Find axial force at this point. Include weight of pilaster.

$$P_u = 0.54k + 0.9\left(0.20\frac{k}{ft}\right)(143.7in)\frac{1ft}{12in} = 2.69k$$

Design for $P_u$ = 2.7 kips, $M_u$ = 361 k-in

# Example: Pilaster Design

Determine *a*

$$a = d - \sqrt{d^2 - \frac{2\left[P_u\left(d - h/2\right) + M_u\right]}{\phi\left(0.8 f'_m b\right)}}$$

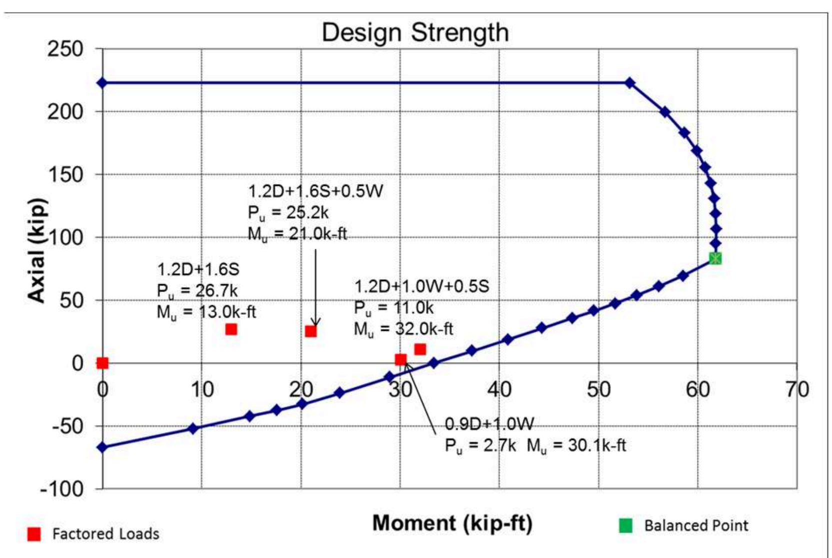
$$= 11.8in - \sqrt{\left(11.8in\right)^2 - \frac{2\left[2.7k\left(11.8in - 15.6in/2\right) + 361k - in\right]}{0.9\left(0.8\right)\left(2.0ksi\right)\left(15.6in\right)}} = 1.50in$$

Determine $A_s$

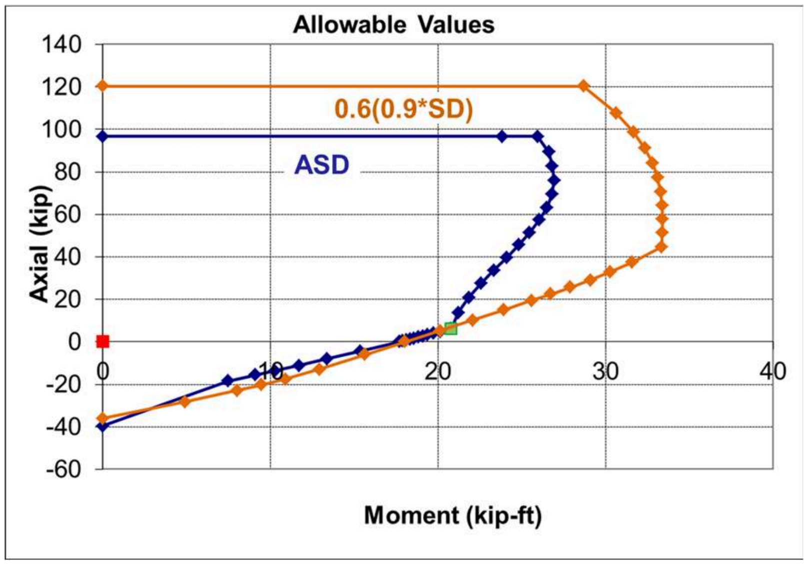
$$A_s = \frac{0.8 f'_m ba - P_u/\phi}{f_y} = \frac{0.8\left(2.0ksi\right)\left(15.62in\right)\left(1.50in\right) - 2.7k/0.9}{60ksi} = 0.57in^2$$

Required steel = 0.57 in$^2$
Use 2-#5 each face, $A_s$ = 0.62 in$^2$
Total bars, 4-#5, one in each cell

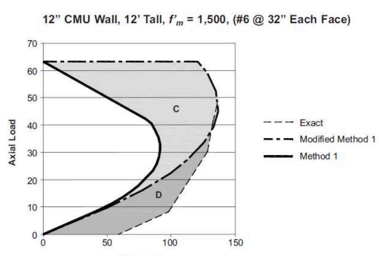
# Example: Pilaster Design

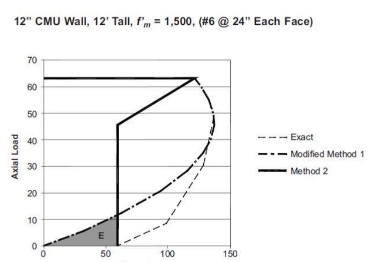


# Example: Pilaster Design




# ASD Design Approximations




Method 1: $\frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1$

Modified Method 1: $f_a + f_b \leq F_b$

Method 2: $F_b' = F_b - f_a$

Reinforced Masonry Engineering Handbook, 7[th] edition

# ASD Design Method

Calculate

$$kd = 3\left[\frac{d}{2} - \sqrt{\left(\frac{d}{2}\right)^2 - \frac{2(P(d-t/2)+M)}{3F_m b}}\right]$$

Is $k \geq k_{bal}$?

For Grade 60 steel

$k_{bal}$ = 0.312

$$k_{bal} = \frac{F_m}{F_m + \frac{F_s}{n}}$$

**YES**

$$A_s = \frac{\frac{F_m (kd) b}{2} - P}{nF_m\left(\frac{1}{k} - 1\right)}$$

Compression controls

**NO**

$$M' = P\left(\frac{t}{2} - \frac{kd}{3}\right)$$

$$A_s = \frac{M - M'}{F_s d\left(1 - \frac{k}{3}\right)}$$

$$\zeta = \frac{(P + A_s F_s)n}{F_s b}$$

$$(kd)_2 = \sqrt{\zeta^2 + 2\zeta d} - \zeta$$

Iterate. Use $(kd)_2$ as new guess and repeat.

Tension controls

# Bearing Walls

- **Second-order effects**

## Walls: Slenderness Effects

1. **Complementary moment method**
   a. Second-order moment directly added by P-δ (9.3.5.4.2)
   b. Usually requires iteration
   c. Difficult for hand calculations for other than simple cases
   d. Basis for second-order analysis in computer programs
   e. Historical method used for masonry design
2. **Second-order analysis**
   a. Added in 2013 TMS 402 Code (9.3.5.4.3)
   b. Computer analysis
3. **Moment magnification method**
   a. Added in 2013 TMS 402 Code (9.3.5.4.3)
   b. Very general, but a bit conservative.

## Walls: 9.3.5.4.2 Complementary Moment

- Assumes simple support conditions.
- Assumes mid-height moment is approximately maximum moment
- Implicit assumption of uniform load and moment at top
- Valid only for the following conditions:
  - $\frac{P_u}{A_n} \leq 0.05 f'_m$ No height limit
  - $\frac{P_u}{A_g} \leq 0.20 f'_m$ height limited by $\frac{h}{t} \leq 30$

**Moment:**

$$M_u = \frac{w_u h^2}{8} + P_{uf}\frac{e_u}{2} + P_u \delta_u$$

$$P_u = P_{uw} + P_{uf}$$

$P_{uf}$ = Factored floor load
$P_{uw}$ = Factored wall load

**Deflection:**

$$\delta_u = \frac{5 M_u h^2}{48 E_m I_n} \qquad M_u < M_{cr}$$

$$\delta_u = \frac{5 M_{cr} h^2}{48 E_m I_n} + \frac{5(M_u - M_{cr})h^2}{48 E_m I_{cr}} \qquad M_u > M_{cr}$$

# Design Procedure: Complementary Moment

Solve simultaneous linear equations:

| $M_u > M_{cr}$ | $M_u \le M_{cr}$ |
|---|---|
| $M_u = \dfrac{\dfrac{w_u h^2}{8} + P_{uf}\dfrac{e_u}{2} + \dfrac{5M_{cr}P_u h^2}{48E_m}\left(\dfrac{1}{I_n} - \dfrac{1}{I_{cr}}\right)}{1 - \dfrac{5P_u h^2}{48E_m I_{cr}}}$ | $M_u = \dfrac{\dfrac{w_u h^2}{8} + P_{uf}\dfrac{e_u}{2}}{1 - \dfrac{5P_u h^2}{48E_m I_n}}$ |
| $\delta_u = \dfrac{\dfrac{5h^2}{48E_m I_{cr}}\left[\dfrac{w_u h^2}{8} + P_{uf}\dfrac{e_u}{2} + M_{cr}\left(\dfrac{I_{cr}}{I_n} - 1\right)\right]}{1 - \dfrac{5P_u h^2}{48E_m I_{cr}}}$ | $\delta_u = \dfrac{\dfrac{5h^2}{48E_m I_n}\left[\dfrac{w_u h^2}{8} + P_{uf}\dfrac{e_u}{2}\right]}{1 - \dfrac{5P_u h^2}{48E_m I_n}}$ |

# Example: Wind Loads

Given: 8 in. CMU wall; Grade 60 steel; Type S masonry cement mortar; $f'_m$=2000psi; roof forces act on 3 in. wide bearing plate at edge of wall.

Required: Reinforcement

Solution:

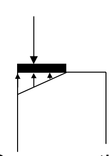
Estimate reinforcement

$$M \sim \frac{wh^2}{8} = \frac{(0.032ksf)(18ft)^2}{8} = 1.30\,\frac{k-ft}{ft}$$

$a$ = 0.24 in..

$A_s$ = 0.078 in$^2$/ft

Try #5 @ 48 in. (0.078 in$^2$/ft)

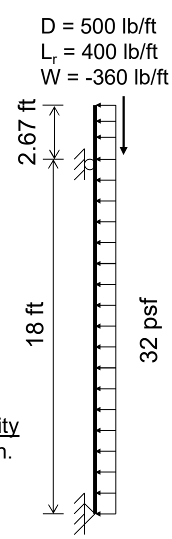


Cross-section of top of wall

Determine eccentricity

e = 7.625in/2 – 1.0 in.

= 2.81 in.



# Example: Wind Loads

Summary of Strength Design Load Combination Axial Forces

(wall weight is 38 psf for 48 in. grout spacing)

| Load Combination | $P_{uf}$ (kip/ft) | $P_{uw}$ (kip/ft) | $P_u$ (kip/ft) |
|---|---|---|---|
| 0.9D+1.0W | 0.9(0.5)+1.0(-0.36) = 0.090 | 0.9(0.038)(2.67+8) = 0.399 | 0.489 |
| 1.2D+1.0W+0.5$L_r$ | 1.2(0.5)+1.0(-0.36) +0.5(0.4) = 0.440 | 1.2(0.038)(2.67+8) = 0.532 | 0.972 |

$P_{uf}$ = Factored floor load; just eccentrically applied load

$P_{uw}$ = Factored wall load; includes wall and parapet weight, found at midheight of wall between supports (9 ft from bottom)

# Example: Wind Loads

Modulus of rupture: use linear interpolation between no grout and full grout

Ungrouted (Type S masonry cement): 51 psi

Fully grouted (Type S masonry cement): 153 psi

$$f_r = 51psi\left(\frac{5\,ungrouted\ cells}{6\,cells}\right) + (153psi)\left(\frac{1\,grouted\ cell}{6\,cells}\right) = 68psi$$

Cracking moment, $M_{cr}$:

Commentary allows inclusion of axial load (9.3.5.4.4)

Use minimum axial load (once wall has cracked, it has cracked)

$$M_{cr} = \frac{(P_u / A_n + f_r)I_n}{t/2} = \frac{\left(\left(489\,\tfrac{lb}{ft}\right)/\left(40.7\,\tfrac{in^2}{ft}\right) + 68\,psi\right)\left(332.0\,\tfrac{in^4}{ft}\right)}{3.81in}$$

$$M_{cr} = 6.97\,\tfrac{k-in.}{ft} = 0.581\,\tfrac{k-in.}{ft}$$

Wall properties determined from NCMA TEK 14-1B Section Properties of Concrete Masonry Walls

# Example: Wind Loads

Modular ratio

$$n = \frac{E_s}{E_m} = \frac{29000ksi}{1800ksi} = 16.1$$

c, depth to neutral axis

Equation 9-31

$$c = \frac{A_s f_y + P_u}{0.64 f'_m b} = \frac{0.0775\frac{in^2}{ft}(60ksi) + 0.489\frac{k}{ft}}{0.64(2.0ksi)12\frac{in}{ft}} = 0.334in$$

$I_{cr}$, cracked moment of inertia

Equation 9-30

$$I_{cr} = n\left(A_s + \frac{P_u}{f_y}\frac{t_{sp}}{2d}\right)(d-c)^2 + \frac{bc^3}{3}$$

$$= 16.1\left(0.0775\frac{in^2}{ft} + \frac{0.489\frac{k}{ft}}{60ksi}\right)(3.812in - 0.334in)^2 + \frac{12\frac{in}{ft}(0.334in)^3}{3}$$

$$= 16.8\frac{in^4}{ft}$$

# Example: Wind Loads

Find $M_u$

$P_{uf}e_u$ is the moment at the top support of the wall; includes eccentric axial load and wind load from parapet.

$$M_{u,top} = P_{uf}e_u - \frac{w_{u,parapet}h^2_{parapet}}{2} = 0.090\tfrac{k}{ft}(2.81in)\tfrac{1ft}{12in} - \frac{0.032ksf(2.67ft)^2}{2} = -0.093\frac{k-ft}{ft}$$

$$M_u = \frac{\frac{w_u h^2}{8} + P_{uf}\frac{e_u}{2} + \frac{5M_{cr}P_u h^2}{48E_m}\left(\frac{1}{I_n} - \frac{1}{I_{cr}}\right)}{1 - \frac{5P_u h^2}{48E_m I_{cr}}}$$

$$= \frac{\frac{0.032ksf(18ft)^2}{8} + \frac{-0.093\frac{k-ft}{ft}}{2} + \frac{5\left(0.581\frac{k-ft}{ft}\right)\left(0.489\frac{k}{ft}\right)(18ft)^2}{48(1800ksi)}\left(\frac{1}{332\frac{in^4}{ft}} - \frac{1}{16.8\frac{in^4}{ft}}\right)\left(\frac{144in^2}{1ft^2}\right)}{1 - \frac{5\left(0.489\frac{k}{ft}\right)(18ft)^2}{48(1800ksi)\left(16.8\frac{in^4}{ft}\right)}\left(\frac{144in^2}{1ft^2}\right)}$$

$$= \frac{(1.296 - 0.046 - 0.043)\frac{k-ft}{ft}}{0.9214} = 1.309\frac{k-ft}{ft}$$

# Example: Wind Loads

Compare to capacity: Commentary 9.3.5.2

*a*, depth of stress block

$$a=\frac{A_s f_y + P_u/\phi}{0.80 f'_m b}=\frac{0.0775\frac{in^2}{ft}(60ksi)+0.489\frac{k}{ft}/0.9}{0.80(2.0ksi)\left(12\frac{in}{ft}\right)}=0.270in$$

$\phi M_n$, design moment

$$\phi M_n=\phi\left(P_u/\phi+A_s f_y\right)\left(d-\frac{a}{2}\right)$$

$$=0.9\left(0.489\frac{k}{ft}/0.9+0.0775\frac{in^2}{ft}(60ksi)\right)\left(3.812in-\frac{0.270in}{2}\right)$$

$$=17.19\frac{k-in}{ft}=1.432\frac{k-ft}{ft}$$

| Load Combination | $M_u$ (kip-ft/ft) | $\phi M_n$ (kip-ft/ft) | $M_u/\phi M_n$ | 2nd Order / 1st Order |
|---|---|---|---|---|
| 1.2D+1.6$L_r$+0.5W | 0.799 | 1.755 | 0.455 | 1.074 |
| 1.2D+1.0W+0.5$L_r$ | 1.416 | 1.574 | 0.900 | 1.097 |
| 0.9D+1.0W | 1.309 | 1.432 | 0.914 | 1.047 |

**OK**

# Example: Wind Loads, Deflections

| Load Combination | D+0.6W | 0.6D+0.6W |
|---|---|---|
| *P* (k/ft) | 0.5+0.6(-0.36) = 0.284 | 0.6(0.5)+0.6(-0.36) = 0.084 |
| $P_w$ (k/ft) | 38(2.67+9) = 0.443 | 0.6(0.443) = 0.266 |
| *P* (k/ft) | 0.727 | 0.350 |
| *c* (in) | 0.350 | 0.325 |
| $I_{cr}$ (in$^4$/ft) | 17.48 | 16.46 |
| $M_{top}$ (k-ft/ft) | -0.002 | -0.049 |
| *M* (k-ft/ft) | 0.805 | 0.767 |
| δ (in) | 0.474 | 0.426 |

Deflection Limit
Section 9.3.5.5

$$\delta_s \le 0.007h = 0.007(18ft)\frac{12in}{ft}=1.51in$$

**OK**

# Example: Wind Loads, Deflections

Deflections, Sample Calculations: **D + 0.6W**
Replace factored loads with service loads

$$\delta = \frac{\dfrac{5h^2}{48E_m I_{cr}}\left[\dfrac{wh^2}{8} + P_f \dfrac{e}{2} + M_{cr}\left(\dfrac{I_{cr}}{I_n} - 1\right)\right]}{1 - \dfrac{5Ph^2}{48E_m I_{cr}}}$$

$$= \frac{\dfrac{5(18ft)^2}{48(1800ksi)\left(17.5\frac{in^4}{ft}\right)}\left[\dfrac{(0.6)(0.032ksf)(18ft)^2}{8} + \dfrac{-0.002\frac{k-ft}{ft}}{2} + 0.581\frac{k-ft}{ft}\left(\dfrac{17.5\frac{in^4}{ft}}{332\frac{in^4}{ft}} - 1\right)\right]\dfrac{144in^2}{1ft^2}}{1 - \dfrac{5\left(0.727\frac{k}{ft}\right)(18ft)^2}{48(1800ksi)\left(17.5\frac{in^4}{ft}\right)}\dfrac{144in^2}{1ft^2}}$$

$$= 0.0393ft = 0.474in.$$

# Example: Maximum Reinforcement

Check Maximum Reinforcement: (Section 9.3.3.2)

- neutral axis is in face shell
- Load combination $D$ + 0.75$L$ +0.525$Q_E$ (9.3.3.5.1 (d))
  - $P_u$ is just dead load = 0.5+0.038(2.67+9) = 0.943k/ft

9.3.3.2 Commentary Equations

$$\rho_{\max} = \frac{A_s}{bd} = \frac{0.64f'_m\left(\dfrac{\varepsilon_{mu}}{\varepsilon_{mu} + \alpha\varepsilon_y}\right) - \dfrac{P_u}{bd}}{f_y}$$

$$\frac{0.64(2.0ksi)\left(\dfrac{0.0025}{0.0025 + 1.5(0.00207)}\right) - \dfrac{0.943\frac{k}{ft}}{12\frac{in}{ft}(3.81in)}}{60ksi} = 0.00917$$

$$\rho = \frac{A_s}{bd} = \frac{0.0775\frac{ft^2}{ft}}{12\frac{in}{ft}(3.81in)} = 0.00169$$

**OK**

## Shear Walls

- **In-plane shear strength**
- **Shear friction**

## Shear Strength: TMS Section 9.3.4.1.2

$V_n = (V_{nm} + V_{ns})\gamma_g$ Equation 9-17

$\gamma_g$ = 0.75 for partially grouted shear walls
$\gamma_g$ = 1.0 otherwise

$\phi = 0.8$

| Shear Span | Maximum $V_n$ | TMS 402 |
|---|---|---|
| $\frac{M_u}{V_u d_v} \leq 0.25$ | $\left(6A_{nv}\sqrt{f'_m}\right)\gamma_g$ | Equation 9-18 |
| $0.25 < \frac{M_u}{V_u d_v} < 1.0$ | $\left(\frac{4}{3}\left(5 - 2\frac{M_u}{V_u d_v}\right)A_{nv}\sqrt{f'_m}\right)\gamma_g$ | Linear interpolation |
| $\frac{M_u}{V_u d_v} \geq 1.0$ | $\left(4A_{nv}\sqrt{f'_m}\right)\gamma_g$ | Equation 9-19 |

# Shear Strength:
# TMS Section 9.3.4.1.2

$$V_{nm} = \left[4.0 - 1.75\left(\frac{M_u}{V_u d_v}\right)\right] A_{nv}\sqrt{f'_m} + 0.25P_u \geq 0$$ Equation 9-20

$M_u/(V_u d_v)$ need not be taken > 1.0
$P_u$ = axial load

$$V_s = 0.5\left(\frac{A_v}{s}\right) f_y d_v$$ Equation 9-21

Vertical reinforcement shall not be less than one-third horizontal reinforcement; reinforcement shall be uniformly distributed, maximum spacing of 8 ft (9.3.6.2)

# Maximum Reinforcement:

Design with Boundary Elements?

**Yes**

Is

$\frac{M_u}{V_u l_w} \leq 1$ OR $\left\{V_u \leq 3A_{nv}\sqrt{f'_m} \text{ AND } \frac{M_u}{V_u l_w} \leq 3\right\}$

AND

$P_u \leq 0.10 A_g f'_m$: Geometrically symmetrical walls

$P_u \leq 0.05 A_g f'_m$: Geometrically unsymmetrical walls

**Yes**: No boundary elements required

**No**: Design boundary elements per TMS 402 Section 9.3.6.6.2

**No**

Design with TMS 402 Section 9.3.3.2.
Area of flexural tensile reinforcement ≤ area required to maintain axial equilibrium under the following conditions
- A strain gradient corresponding to $\varepsilon_{mu}$ in masonry and $\alpha f_y$ in tensile reinforcement
- Axial forces from loading combination $D$ + 0.75$L$ + 0.525$Q_E$ .
- Compression reinforcement, with or without lateral restraining reinforcement, can be included.

Is $M_u/(V_u d_v) \geq 1$ ?

**Yes**

Ordinary reinforced walls: α = 1.5
Intermediate reinforced walls: α = 3
Special reinforced walls: α = 4

**No**

α = 1.5

## Maximum reinforcement: TMS 402 Sec. 9.3.3.2

Three methods for checking maximum reinforcement

- Commentary equations
  - only applicable for certain cases
- Determine location of neutral axis based on specified strain condition
  - Find axial capacity and check that axial force from $D + 0.75L + 0.525Q_E$ is less than axial capacity
- Determine location of neutral axis for given axial force, compute strain in extreme tension steel, and compare to minimum strain
  - Usually requires using trial and error to find the location of the neutral axis

## Shear Friction Strength: TMS 402 Sec. 9.3.6.5

$M_u/(V_u d_v) \leq 0.5 \qquad V_{nf} = \mu\left(A_{sp} f_y + P_u\right)$

$M_u/(V_u d_v) \geq 1.0 \qquad V_{nf} = 0.42 f'_m A_{nc}$

Linear interpolation for intermediate values

$A_{nc}$ = area of masonry in compression at nominal moment capacity

$A_{sp}$ = reinforcement within net shear area

Coefficient of friction

- $\mu$ = 1.0 for masonry on concrete with unfinished surface, or concrete with a surface that has been intentionally roughened
- $\mu$ = 0.70 for all other conditions
- UBC (1997) required concrete abutting structural masonry to be roughened to a full amplitude of 1/16 inch.

## Example:
## Ordinary Reinforced Shear Wall

Given: 10 ft high x 16 ft long 8 in. CMU shear wall; Grade 60 steel, Type S mortar; $f'_m$ = 2000 psi; superimposed dead load of 1 kip/ft. In-plane seismic load of 100 kips. $S_{DS}$ = 0.4

Required: Design the shear wall; ordinary reinforced shear wall

Solution: Check using 0.9D+1.0E load combination.

- Try 2-#5 at end and #5 @ 40 in.

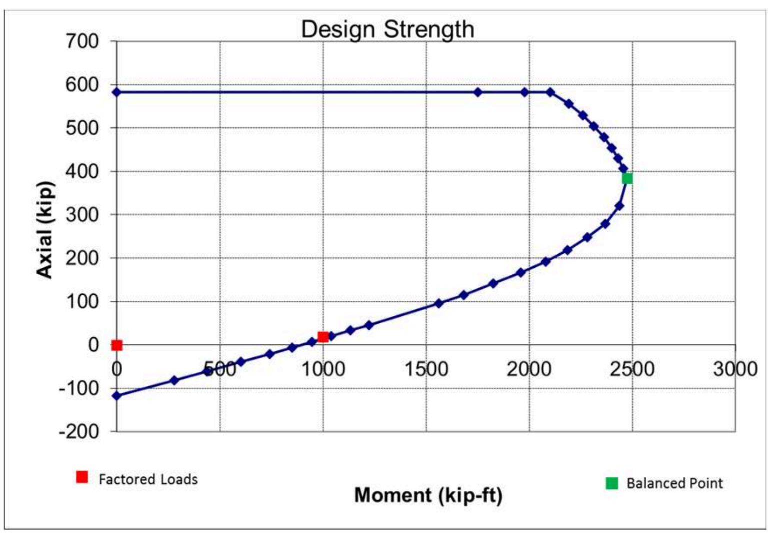
Weight of wall: 40 psf(10ft)(16ft) = 6400 lb

Lightweight units, grout at 40 in. o.c. 40 psf

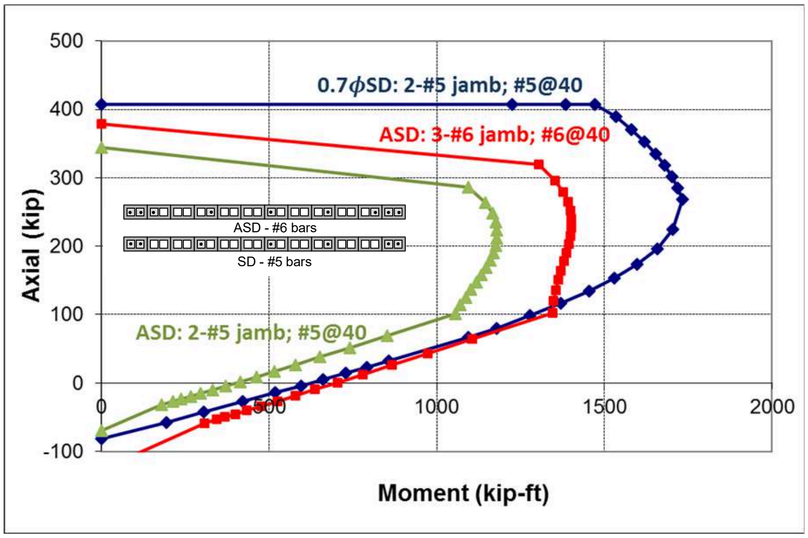
$P_u$ = [0.9 - 0.2($S_{DS}$)]$D$ = [0.9 - 0.2(0.4)](1k/ft(16ft)+6.4k) = 18.4 kips

From interaction diagram OK; stressed to 97% of capacity

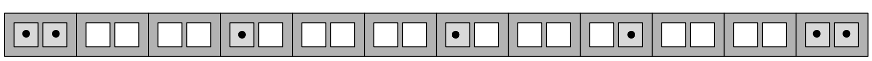
## Example:
## Ordinary Reinforced Shear Wall



## Example:
## Ordinary Reinforced Shear Wall




## Example:
## Ordinary Reinforced Shear Wall



Calculate net area, $A_{nv}$, including grouted cells.

$$A_{nv} = 2.5in(192in) + 7(8in)(7.62in - 2.5in) = 767in^2$$

Maximum $V_n$

$$\phi V_n = \phi\left[\frac{4}{3}\left(5 - 2\frac{M_u}{V_u d_v}\right)A_{nv}\sqrt{f'_m}\right]\gamma_g =$$

$$0.8\left[\frac{4}{3}(5 - 2(0.625))(767in^2)\sqrt{2000psi}\right]0.75 = 102.9kips$$

**OK**

## Example: Ordinary Reinforced Shear Wall

Top of wall (critical location for shear): $P_u = (0.9\text{-}0.2S_{DS})D = 0.82(1\text{k/ft})(16\text{ft}) = 13.1$ kips

Determine $\phi V_{nm}$

$$\phi V_{nm} = \phi\left[\left(4.0 - 1.75\left(\frac{M_u}{V_u d_v}\right)\right)A_{nv}\sqrt{f'_m} + 0.25P_u\right]$$

$$= 0.8\left[(4.0 - 1.75(0.625))(767in^2)\sqrt{2000\,psi}\,\tfrac{1kip}{1000lb} + 0.25(13.1k)\right] = 82.4kips$$

Required steel strength

$$V_{ns} = \frac{V_u}{\phi\gamma_g} - \frac{\phi V_{nm}}{\phi} = \frac{100k}{0.8(0.75)} - \frac{82.4k}{0.8} = 63.7k$$

Use #5 bars in bond beams. Determine spacing.

$$V_{ns} = 0.5\left(\frac{A_v}{s}\right)f_y d_v \quad \Rightarrow$$

$$s = \frac{0.5A_v f_y d_v}{V_{ns}} = \frac{0.5(0.31in)(60ksi)(192in)}{63.7k} = 28.0in$$

$s$ ≤ min{$d$/2, 48 in.} = min{94 in., 48 in.} = 48 in. Code 8.3.5.2.1
In strength design, this provision only applies to beams (9.3.4.2.3 (e))
Suggest that minimum spacing also be applied to shear walls.

**Use #5 at 24 in. o.c.**

## Example: Ordinary Reinforced Shear Wall

Shear friction: Specify an unfinished surface; $\mu$ = 1.0

Shear ratio $M_u/(V_u d_v) = (V_u h)/(V_u d_v) = 120in./192in. = 0.625$

Since $0.5 < M_u/(V_u d_v) < 1.0$ use linear interpolation

Area of reinforcement crossing shear plane, $A_{sp}$ $A_{sp} = 7(0.31in.^2) = 2.17in.^2$

For $M_u/(V_u d_v) \le 0.5$ $V_{nf} = \mu(A_{sp} f_y + P_u) = 1.0(2.17in.^2(60ksi) + 18.4k) = 148.6kip$

# Example:
# Ordinary Reinforced Shear Wall

For $M_u/(V_u d_v) \geq 1.0$ From interaction diagram (spreadsheet), $c$ = 11.7 in. @ $\phi P_n$ = 18.8k

$$V_{nf} = 0.42 f'_m A_{nc} = 0.42(2ksi)(7.625in.)(11.7in.) = 74.9kip$$

Linear Interpolation

$$V_{nf} = 74.9k + \frac{1.0-0.625}{1.0-0.5}(148.6k - 74.9k) = 130.2kip$$

Design Strength

$$\phi V_{nf} = 0.8(130.2kip) = 104.1kip \geq V_u = 100kip$$

**OK**

# Special Reinforced Shear Walls

1. Maximum spacing of vertical and horizontal reinforcement is min{1/3 length of wall, 1/3 height of wall, 48 in. [24 in. for masonry in other than running bond]}.
2. Minimum area of vertical reinforcement is 1/3 area of shear reinforcement
3. Shear reinforcement anchored around vertical reinforcing with standard hook
4. Sum of area of vertical and horizontal reinforcement shall be 0.002 times gross cross-sectional area of wall
5. Minimum area of reinforcement in either direction shall be 0.0007 times gross cross-sectional area of wall [0.0015 for horizontal reinforcement for masonry in other than running bond].

## Special Reinforced Shear Walls: Shear Capacity Design

Minimum shear strength (7.3.2.6.1.1):

- Design shear strength, $\phi V_n$, greater than shear corresponding to 1.25 times nominal flexural strength, $M_n$
- Except nominal strength, $V_n$, need not be greater than $2.5V_u$.

Normal design: $\phi V_n$ has to be greater than $V_u$. Thus, $V_n$ has to be greater than $V_u/\phi$ = $V_u/0.8$ = $1.25V_u$. This requirement doubles the shear.

## Example: Special Reinforced Shear Wall

Given: 10 ft high x 16 ft long 8 in. CMU shear wall; Grade 60 steel, Type S mortar; $f'_m$ = 2000 psi; superimposed dead load of 1 kip/ft. In-plane seismic load (from ASCE 7-10) of 100 kips. $S_{DS}$ = 0.4

Required: Design the shear wall; **special reinforced shear wall**

Solution: Check using 0.9D+1.0E load combination.

- Shear capacity design provisions (Section 7.3.2.6.1.1)
  - $\phi V_n \geq$ shear corresponding to $1.25M_n$.
    - Minimum increase is 1.25/0.9 = 1.39
  - $V_n$ need not exceed $2.5V_u$
    - Normal design $V_n \geq V_u/\phi = V_u/0.8 = 1.25V_u$
    - Increases shear by a factor of 2
- **Fully grout wall** (Max $\phi V_n$ was 103 kips)

$$A_{nv} = 7.625in.(192in.) = 1464in^2$$

# Example: Special Reinforced Shear Wall

- Previously, $\phi M_n$ = 1028 k-ft; $M_n$ = 1142 k-ft ($P_u$ = 18.4k)
  - $1.25M_n$ = 1428 k-ft; Design for 143 kips
- But wait, wall is fully grouted. Wall weight has increased to 75 psf
  - For $P_u$ = 23.0k, fully grouted, $M_n$ = 1178 k-ft, $1.25M_n$ = 1474 k-ft
  - Design for 147 kips
- But wait, need to check load combination of 1.2D + 1.0E
  - $P_u = [1.2 + 0.2(S_{DS})]D$ = 35.8 kips, $1.25M_n$ = 1601 k-ft
  - Design for 160 kips
- Bottom line: any change in wall will change $M_n$, which will change design requirement; also need to consider all load combinations
  - Often easier to just use $V_n = 2.5V_u$.

# Example: Special Reinforced Shear Wall

Determine $\phi V_{nm}$

$$\phi V_{nm} = \phi\left[\left(4.0 - 1.75\left(\frac{M_u}{V_u d_v}\right)\right)A_{nv}\sqrt{f'_m} + 0.25P_u\right]$$

$$= 0.8\left[(4.0 - 1.75(0.625))(1464in^2)\sqrt{2000psi}\,\tfrac{1kip}{1000lb} + 0.25(13.1k)\right] = 154.8kips$$

Required steel strength

$$V_{ns} = \frac{V_u - \phi V_{nm}}{\phi} = \frac{160.1k - 154.8k}{0.8} = 6.6k$$

Using shear from $1.25M_n$

Use #5 bars in bond beams.
Determine spacing.

$$V_{ns} = 0.5\left(\frac{A_v}{s}\right)f_y d_v \quad \Rightarrow$$

$$s = \frac{0.5A_v f_y d_v}{V_{ns}} = \frac{0.5(0.31in)(60ksi)(192in)}{6.6k} = 269in$$

7.3.2.6 (b) Maximum spacing of reinforcing of 1/3 length, 1/3 height, or 48 in.
Use maximum spacing of 1/3(height) = 40 in.

$\boxed{\textbf{Use \#5 at 40 in. o.c.}}$

Using shear from $V_n = 2.5V_u$, $s$ = 31.6 inch; use #5 @ 32 in. o.c.

# Example:
# Special Reinforced Shear Wall

Section 9.3.3.2 Maximum Reinforcement

Since $M_u/(V_u d_v) < 1$, strain gradient is based on $1.5\varepsilon_y$.

| Strain | c/d, CMU | c/d, Clay |
|---|---|---|
| $1.5\varepsilon_y$ | 0.446 | 0.530 |
| $3\varepsilon_y$ | 0.287 | 0.360 |
| $4\varepsilon_y$ | 0.232 | 0.297 |

$c$ = 0.446(188in.) = 83.8 in.

- Calculate axial force based on c = 83.8 in.
- Include compression reinforcement
- $\phi P_n$ = 726 kips
- Assume a live load of 1 k/ft
- $D + 0.75L + 0.525Q_E$ = (1k/ft + 0.75(1k/ft))16ft = 28 kips **OK**

# Example:
# Special Reinforced Shear Wall

- Section 9.3.6.5: Maximum reinforcement provisions of 9.3.3.2 do not apply if designed by this section (boundary elements)
- Special boundary elements not required if:

| | |
|---|---|
| $P_u \le 0.1 f'_m A_g$ | geometrically symmetrical sections |
| $P_u \le 0.05 f'_m A_g$ | geometrically unsymmetrical sections |

**AND**

$\frac{M_u}{V_u d_v} \le 1$ OR $\boxed{V_u \le 3A_n\sqrt{f'_m} \text{ AND } \frac{M_u}{V_u d_v} \le 3}$

For our wall, $M_u/V_u d_v < 1$ **OK**

$P_u < 0.1 f'_m A_g$ = 0.1(2.0ksi)(1464in$^2$) = 293 kips

## Summary

- Strength design in masonry similar to reinforced concrete
- Strength design provides more efficient use of reinforcement when:
  - Allowable masonry stress controls
  - Large dead load
  - Shear walls with distributed reinforcement
- Issues with strength design
  - All walls subjected to out-of-plane loads need to be designed for second-order effects
  - Maximum reinforcement requirements sometimes control
    - Designers switch to ASD, which then requires more steel. This makes no sense.

This concludes The American Institute of Architects Continuing Education Systems Course

The Masonry Society

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